Abstract: This paper develops a two-country, two-sector model of trade where the only difference between two countries is the cost of human capital formation. It is shown that this difference completely shapes the pattern of trade. Trade, in turn, affects the distribution of human capital both at extensive and at intensive margins, income distribution, and welfare in each country. Since not all agents gain from trade, the paper also investigates the conditions under which trade between two countries becomes possible if the final decision in each country is based on majority voting. Finally, the paper shows that lowering the cost of human capital in one country has asymmetric effects on human capital formation and the income inequality between skilled and unskilled workers across countries.

Keywords: human capital, income inequality, majority voting, trade

JEL Classification: D7, F1, L1, O1

DOI 10.1515/bejeap-2014-0041

1 Introduction

The standard Heckscher–Ohlin trade theory emphasizes the differences in factor endowments across countries as determinant of trade. Since this theory takes the aggregate factor endowments as exogenously fixed, trade has no impact on factor endowments. However, this contradicts the common belief that trade affects not only sectoral composition but also human capital formation across countries. For example, Edmonds, Pavcnik, and Topalova (2010) find that India’s 1991 tariff reform reduced schooling in districts where employment concentrated in industries protected by high tariffs.\(^1\) Furthermore, some recent

---

\(^1\) In another study, Bustos (2011) finds that after a reduction in Brazil’s tariffs, the most productive Argentinean firms upgrade skill, while the least productive ones downgrade.

Bulent Unel, Department of Economics, Louisiana State University, Baton Rouge, LA 70803, USA, E-mail: bunel@lsu.edu
studies have shown that the distribution of human capital across workers can also be an important source of comparative advantage even though countries may have similar aggregate endowments. Bombardini, Gallipoli, and Pupato (2012) using data from the International Adult Literacy Survey show that skill dispersion has significant effect on trade flows.

This paper develops a two-country, two-sector competitive model of trade to study the interaction between human capital and trade. Labor is the only factor of production, and one sector (agriculture) uses only unskilled labor, whereas the other sector (manufacturing) uses workers with varying human capital (skill). Workers in agriculture earn the same wage rate, but the earnings of workers in manufacturing are proportional to their skill. The decision to become a skilled worker is endogenous, and the cost of human capital acquisition depends on individuals' innate ability (characterized by a common distribution) as well as country-specific factors. The two countries are symmetric in every aspect except for the differences in the costs of human capital formation.

The model has several interesting results. First, it shows that cross-country differences in the costs of human capital acquisition shape the pattern of trade: under free trade the country with the lower cost of human capital acquisition (Home) exports the manufacturing good, whereas the other country (Foreign) exports the agricultural good. However, unlike traditional models, trade in turn has an effect on the distribution of human capital in each country: the fraction of individuals who choose to acquire human capital and their per capita level of human capital increase in Home, whereas they decrease in Foreign. Second, the impact of trade on the between-group and within-group income inequalities depend on the distribution of ability levels. Free trade, however, always improves the aggregate welfare in both countries. Since not all agents gain from trade, this paper also investigates the conditions under which trade between the two countries becomes possible if the final decision in each country is based on majority voting. It turns out that two countries choose to trade if demand for the manufacturing good is strong and the cross-country difference in the cost of human capital is sufficiently high. Finally, the model is also suitable to study the effects of a unilateral change in the cost of human capital. A reduction in the cost of human capital in Foreign, for example, increases human capital both at extensive and at intensive margins and improves aggregate welfare in Foreign; while having opposite effects on these variables in Home.

In an influential paper, Findlay and Kierzkowski (1983) incorporate human capital formation into the two-factor, two-good model of trade to study patterns of trade. In their model, individuals acquire human capital over time through education which uses physical capital and time. This paper differs from theirs in
three aspects. First, although their model highlights the interaction between human capital formation and trade, the pattern of trade is ultimately driven by the cross-country differences in the aggregate capital-labor ratio as in the standard trade theory. This paper focuses on the distributional aspects of the cost of human capital formation under the assumption that aggregate endowments are the same across countries. Second, in their model, those who become skilled workers have the same level of human capital and earn the same wage; whereas in my model, individuals who become skilled workers vary in their human capital and wages. Finally, the present paper conducts an extensive welfare analysis and identifies conditions under which a political-economy equilibrium emerges.

Bond (1986) presents a small open economy model where firms are heterogeneous due to differences in managerial ability as in Lucas (1978) to study the extent to which the results of the standard trade theorems hold. Ishikawa (1996) incorporates workers’ heterogeneity in the form of different human capital into the Heckscher-Ohlin model to examine patterns of trade and migration. Although individuals choose to become skilled or unskilled workers in these models, individuals don’t choose to acquire human capital, and thus trade does not have any impact on human capital formation. In addition, countries in these models differ in aggregate endowments.

Cartiglia (1997) develops a two-sector small open economy model where individuals can either go to school and become skilled workers or work as unskilled workers. In his model, only wealthy individuals can afford to go to school, because credit markets are imperfect and individuals differ in the amount of physical capital that they have. Ranjan (2001) proposes a small open economy model to study the effect of credit-market imperfections on human capital investment. Like Cartiglia (1997), in his model the impact of trade on human capital accumulation depends on the interaction between the distribution of wealth and credit-market imperfections.

This paper is related to the recent literature that emphasizes the differences in distributions of factor endowments in explaining the pattern of comparative advantage. Grossman and Maggi (2000) present a model of trade between countries with similar aggregate factor endowments to study how the differences in dispersions of skill across the workers of the two countries determine the

2 Borsook (1987) extends the Findlay and Kierzkowski model by considering individuals with different abilities. However, as in the Findlay and Kierzkowski model, differences in the aggregate factor endowments across countries determine the pattern of trade.

3 See also Chesnokova and Krishna (2009) and Bonfatti and Ghatak (2013) who investigate the impact of trade on skill acquisition in the presence of capital-market imperfections.
pattern of trade. Bougheas and Riezman (2007) also show that the differences in human capital distributions can determine the pattern of trade between two otherwise symmetric countries. However, unlike the present paper, in these studies the distributions of human capital are exogenous.

This paper is also related to the literature that studies the impact of trade on growth and productivity. Eicher (1999) develops a small open economy model where both the supply of skilled workers and technical progress are endogenously determined. He shows that trade in goods can increase the supply of skilled workers as well as the growth rate of technology. By analyzing a small open economy model, the source of comparative advantage is not identified in his model. Dinopoulou and Segerstrom (1999) develop a Schumpeterian growth model with trade where individuals choose to become skilled workers. They show that trade increases the skill-premium and results in skill upgrading. In their model, countries are symmetric, product markets are imperfectly competitive, and skilled workers earn the same wage. Galor and Mountford (2008) also study the dynamic interaction between trade and human capital. They argue that trade has been the major factor behind the distribution of income, human capital, and population across the countries. The cross-country differences in production technologies and parents’ investment in their children’s education play key roles in their model. In my model, countries use the same the production technology in each sector and individuals acquire human capital by themselves.

The rest of this paper is organized as follows. Section 2 introduces the model and studies the equilibrium in autarky. Section 3 characterizes the equilibrium in open economy, and investigates inequality and welfare effects of trade. It also studies the impact of a change in the cost of human capital formation in one country on inequality and welfare in each country. Section 4 concludes.

2 Setup of the model

I begin by considering the equilibrium analysis under autarky to highlight the main points of the model. Consider a country that produces two goods, an agricultural good ($a$) and a manufacturing good ($m$) using labor as the only

---

4 Yeaple (2005) studies how the interaction between technology adoption, trade, and workers’ heterogeneity gives rise to firm heterogeneity. In his model, countries are symmetric, firms compete with each other in a monopolistic fashion, and increasing returns to scale is the driving factor for trade.
factor of production.\(^5\) There is a continuum of individuals with constant mass of one \((L = 1)\), each having different ability to acquire human capital. Ability levels (indexed by \(\theta\)) are drawn from a common distribution \(g(\theta)\) with cumulative distribution \(G(\theta)\). The supply of skilled and unskilled labor is endogenously determined based on individuals ability levels and country-specific factors.

### 2.1 Demand

Individuals have identical preferences over the two goods as described by the following utility function

\[
u = \left(\frac{c_a}{1 - \beta}\right)^{1-\beta} \left(\frac{c_m}{\beta}\right)^{\beta},\]

where \(c_i\) denotes consumption of good \(i = a, m\); and \(\beta \in (0, 1)\) is an exogenous, constant parameter.

Denote by \(e\) the income of the individual, the utility maximization problem then yields

\[
c_a = (1 - \beta)e/p_a, \quad c_m = \beta e/p_m.\]

In the subsequent analysis, the agricultural good is chosen as numeraire so that its price equals one (i.e., \(p_a = 1\)), and for notational simplicity, the price of manufacturing good is denoted by \(p\). By choosing the agricultural good as numeraire, all other variables are measured in terms of this good. Substituting the above consumption levels into [1] yields the following indirect utility function

\[
v = p^{-\beta}e.\]

### 2.2 Production

The agricultural sector uses unskilled labor, and one unit of (unskilled) labor is required to produce one unit of agricultural good. Since this good is chosen as numeraire, the production technology ensures that the wage rate of unskilled labor equals one, i.e., \(w = 1\).

Workers in the manufacturing sector are heterogeneous with respect to their human capital. Production technology is constant returns to scale such that a

\(^5\) Physical capital is assumed away for the sake of simplicity. Ferguson (1978) shows that when capital is fully mobile the pattern of comparative advantage is determined by the differences in labor productivity. Thus, extending the model by incorporating fully mobile capital will not change the main results.
worker with $h$ units of human capital produces $h$ units of good $m$. Each worker is also the owner of his own production, and thus the revenue (gross income) generated by a worker with human capital $h$ is $ph$. However, acquisition of human capital is costly. The cost of human capital acquisition of a worker with ability $\theta$ is

$$z(h, \theta) = \frac{bh^2}{2\theta}. \quad [4]$$

measured in terms of the numeraire good. The parameter $b > 0$ represents a country-specific, exogenous constant.\(^6\)

Since each individual’s welfare increases with their income, workers wishing to acquire human capital maximize their net income, $e(\theta) = ph - z(h; \theta)$, by setting

$$h = \frac{\theta p}{b}. \quad [5]$$

This equation indicates that more able individuals acquire more human capital. Furthermore, the human capital level decreases in $b$ and increases in $p$. Substituting $h$ from eq. [5] into $e(\theta) = ph - z(h; \theta)$ yields

$$e(\theta) = \frac{\theta p^2}{2b}. \quad [6]$$

Thus, the income of an individual working in manufacturing sector increases in his ability $\theta$ and the price $p$, decreases in the cost parameter $b$.

Given that labor is mobile between the two sectors, an individual chooses to work in manufacturing if and only if $e(\theta) \geq w = 1$. The critical ability level, $\theta^*$, at which an individual is indifferent for working between the two sectors is determined by $e(\theta^*) = 1$. Hence,

$$\theta^* = \frac{2b}{p^2}. \quad [7]$$

Any individual with $\theta \geq \theta^*$ invests in human capital and works in manufacturing sector, while individuals with $\theta < \theta^*$ work in agriculture and earns $e(\theta) = 1$. According to eq. [7], the ability cutoff $\theta^*$ increases in the cost parameter $b$; decreases in the relative price of manufacturing $p$.

\(^6\) That is, $y_m(h) = h$. Assuming that $y_m(h) = h^{\eta l^{1-\eta}}$ so that the manufacturing sector uses both skilled and unskilled workers does not change results. However, when both sectors use both skilled and unskilled workers, the model becomes analytically intractable and possibly yields multiple equilibria which are hard to interpret.

\(^7\) Considering a more general cost function $z = [f + bh^2/2]p^\eta$ (where $f$ is a fixed parameter and $\eta \in (0, 1)$ is a constant) so that the investment in human capital uses both goods, makes exposition more complicated without changing the main results.
The total output produced by each sector then is

\[ Y_a = G(\theta^*), \]  

[8a]

\[ Y_m = \int_{\theta^*}^{\infty} h(\theta) g(\theta) d\theta = \mathbb{E}[\theta > \theta^*]p/b, \]  

[8b]

where \( \mathbb{E}[\theta > \theta^*] = \int_{\theta^*}^{\infty} \theta g(\theta) d\theta \) (and recall that the supply of labor is normalized to one). Furthermore, substituting \( h \) from eq. [5] into the cost function [4] yields \( z(\theta) = \theta p^2/2b = \theta/\theta^* \), where the last equality follows from eq. [7]. Aggregating over the corresponding individuals yields the total amount of agricultural good used in the human capital formation:

\[ Z_a = \frac{\mathbb{E}[\theta > \theta^*]}{\theta^*}. \]  

[9]

Given the cutoff \( \theta^* \), the income distribution and welfare across individuals can easily be characterized. Substituting \( p \) from eq. [7] into the income per capita eq. [6] implies that \( e(\theta) = \theta/\theta^* \) for all \( \theta > \theta^* \). Thus, the per capita income (or expenditure) is

\[ e(\theta) = \begin{cases} 
1 & \text{if } \theta < \theta^* \\
\theta/\theta^* & \text{if } \theta \geq \theta^* 
\end{cases} \]  

[10]

The aggregate expenditure, \( E \), then is

\[ E = \int e(\theta) g(\theta) d\theta = G(\theta^*) + \frac{\mathbb{E}[\theta > \theta^*]}{\theta^*}. \]  

[11]

Finally, using eq. [10] in the indirect utility function [3] yields the distribution of welfare across individuals

\[ v(\theta, p) = \begin{cases} 
p^{-\beta} & \text{if } \theta < \theta^* \\
p^{-\beta} \theta/\theta^* & \text{if } \theta \geq \theta^* 
\end{cases} \]  

[12]

Since each individual’s welfare depends on his own income, the way aggregate (social) welfare is measured becomes important in investigating the impact of trade on welfare. The aggregate welfare is measured as the sum of individuals’ utilities.\(^8\)

---

\(^8\) The welfare analysis based on the welfare function [12] is permissible if the economy admits a representative agent. Note that \( v(p, e) = e p^{-\beta} \) implies that the indirect utility function of each individual is in the Gorman form; as a result, individuals’ preferences can be aggregated and represented by those of a representative agent. See chapter 4 in Mas-Colell, Winston, and Green (1995) for more on the aggregation of preferences.
\[ V = \int v(\theta, p)g(\theta) d\theta = p^{-\beta} E, \]  
where the last equality directly follows from eqs [10] and [11].

2.3 Closed economy equilibrium

In autarky, \( C_a = Y_a - Z_a \) and \( C_m = Y_m \), where \( C_i \) is the aggregate consumption of good \( i = a, m \). Equation [2] then implies that \( p Y_m / (Y_a - Z_a) = \beta / (1 - \beta) \).

Substituting \( Z_a \) from eq. [9] into the latter and using eq. [7] yields

\[ G(\theta^*) = \frac{2 - \beta}{\beta} E[\theta \geq \theta^*]. \]

The left-hand side (LHS) is increasing in \( \theta^* \), whereas the right-hand side (RHS) is decreasing in it. Thus, there exists a unique cutoff \( \theta^* \) that solves eq. [14]. In addition, the ability cutoff level \( \theta^* \) is independent from the cost parameter \( b \).

**Lemma 1** There exists a unique ability cutoff level \( \theta^* \) that satisfies eq. [14].

Consider now two countries (Home and Foreign) that are identical in terms of preferences, production technologies, and the distributions of ability. However, the cost parameter \( b_j \) \((j = H, F)\) differs across countries. Note that the equilibrium condition [14] indicates that both countries have the same ability cutoff in autarky, i.e., \( \theta^*_H = \theta^*_a = \theta^*_F \) (with superscript \( a \) denoting autarky). Assuming that \( b_H < b_F \), it then follows from eq. [7] that the relative price of manufacturing good in autarky is cheaper in Home, i.e., \( p^a_H < p^a_F \). This difference in the relative price of the manufacturing good across countries creates an opportunity to trade.

3 The open economy

Consider a world of two countries \((j = H, F)\) which are symmetric in every aspect except for the cost parameter \( b_j \). Without loss of generality, assume that \( b_H < b_F \) so that (ceteris paribus) the cost of human capital acquisition is cheaper in Home. The cross-country differences in the costs of human capital formation can arise from the differences in the quality of institutions, the access to credit and training, labor market frictions, and so on. This section investigates the impact of trade between these two countries on human capital formation, income distribution, and welfare in each country.
Trade between the two countries is free\(^9\) and this ensures that both countries face the same price for each good. In addition, workers in the agricultural sector earn the same wage across both countries, i.e., \(w_H = w_F = 1\). The model is closed with two additional assumptions. First, trade is balanced:

\[
(Y_{aj} - Z_{aj} - C_{aj}) + p(Y_{mj} - C_{mj}) = 0, \quad j = H, F,
\]

where \(Z_{aj}\) is given by eq. [9]. Second, the global demand for each good \((i = a, m)\) must be equal to its (net) global supply:

\[
C_{iH} + C_{iF} = Y_{iH} + Y_{iF} - I_i(Z_{aH} + Z_{aF}),
\]

where \(I_i\) is an indicator variable that equals one if \(i = a\), and zero otherwise.

### 3.1 Open economy equilibrium

Note that the balanced trade condition [15] implies that \(E_j = C_{aj} + pC_{mj} = Y_{aj} - Z_{aj} + pY_{mj}\). Using eqs [8a], [8b], and [9] then implies that \(E_j\) is still given by eq. [11] with \(\theta^*\) is replaced by \(\theta^*_j\).

To determine the cutoff level \(\theta^*_j\), the market clearing condition [16] will be used. Substituting \(C_{aj} = (1 - \beta)E_j\) and \(Y_{aj} - Z_{aj}\) into eq. [16] for \(i = a\), and rearranging the terms yields

\[
\sum_j G(\theta^*_j) = \frac{2 - \beta}{\beta} \sum_j \frac{\mathbb{I}[\theta \geq \theta^*_j]}{\theta^*_j}.
\]

Since \(\theta^*_j\) is a decreasing function of \(p\) as indicated by eq. [7], the LHS is decreasing in \(p\), whereas the RHS is increasing in it. Thus, the above equation yields a unique solution for \(p\). Substituting this back into eq. [7] then implies a unique solution for \(\theta^*_j\). In addition, since it is assumed that \(b_H < b_F\), it then follows that \(\theta^*_H < \theta^*_F\). The following lemma summarizes these results.

**Lemma 2** Assume that \(b_H < b_F\). There exist unique ability cutoffs \(\theta^*_H\) and \(\theta^*_F\) that satisfy the equilibrium condition [17]. In addition, under free trade Home’s ability cutoff is smaller than Foreign’s, i.e., \(\theta^*_H < \theta^*_F\).

---

\(^9\) One can also introduce variable trade costs into the model. If these costs are of an iceberg nature, the main results remain qualitatively the same. However, if these costs represent tariffs, then one has to specify the objective of the government in each country. For example, revenues generated by tariffs can be used to finance costs of human capital formation or distributed to unskilled workers to reduce the income inequality between skilled and unskilled workers. Analyzing the model under each case is an interesting exercise and is left for future research.
How do these cutoff levels compare to those in autarky? As discussed in the previous section, the ability cutoffs in autarky are the same in both countries. Since $\theta^*_H/C^3 < \theta^*_F/C^3$ under free trade, it then follows that trade liberalization decreases the ability cutoff in Home, while increasing it in Foreign. This further suggests that in equilibrium Home should export manufacturing good, and Foreign agricultural good. This is indeed the case, and to see this consider the net supply of agricultural good $Y_{aj} - Z_{aj} - C_{aj}$. Using eqs [2], [8a], [9] and [11] implies

$$Y_{aj} - Z_{aj} - C_{aj} = \beta G(\theta^*_j) - (2 - \beta) \frac{\mathbb{E}[\theta > \theta^*_j]}{\theta^*_j}. \quad [18]$$

Using $\theta^*_H/C^3 < \theta^*_F/C^3$ in eq. [18] then implies that $Y_{ah} - Z_{ah} - C_{ah} < Y_{af} - Z_{af} - C_{af}$. This combined with the market clearing condition [16] implies that $Y_{ah} - Z_{ah} - C_{ah} < 0$ and $Y_{af} - Z_{af} - C_{af} > 0$. That is, Home exports the manufacturing good and imports the agricultural good. Thus, the pattern of trade is determined by the differences in the costs of human capital acquisition.

The result that $\theta^*_H/C^3 < \theta^*_F/C^3$ (or $\theta^*_H/C^3 > \theta^*_F/C^3$) indicates that trade liberalization increases (decreases) the fraction of individuals who acquire human capital in Home (Foreign). Furthermore, since the relative price of the manufacturing good increases after trade liberalization, eq. [5] implies that the amount of human capital acquired by each individual in Home is greater than that in autarky. The opposite happens in Foreign. Thus, trade liberalization increases (decreases) human capital in Home (Foreign) both at extensive and at intensive margins.

**Proposition 1** Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. The only difference between the two countries is their costs of human capital acquisition, and assume that (ceteris paribus) the cost is cheaper in Home (i.e., $b_H < b_F$). Moving from autarky to free trade

- **a.** lowers the ability cutoff level in Home, raises it in Foreign (i.e., $\theta^*_H \downarrow, \theta^*_F \uparrow$);
- **b.** makes Home export manufacturing good, and Foreign export agricultural good;
- **c.** increases human capital in Home both at extensive and intensive margins, while having an opposite effect on it in Foreign.

The intuition behind these results is simple. Exposure to trade increases the relative price of the manufacturing good in Home, while decreasing it in Foreign. This makes manufacturing more (less) attractive in Home (Foreign), and thereby inducing more (less) individuals to work in this sector. Since individuals have identical preferences across countries, the expansions in Home’s manufacturing and Foreign’s agriculture makes Home export manufacturing good and Foreign...
export agricultural good. In addition, increased income in Home’s manufacturing induces workers in this sector to acquire more human capital.

Cross-country differences in the cost of human capital acquisition make human capital differ across countries, which in turn determines the pattern of trade presented in Proposition 1. The importance of the distribution of human capital on the pattern of trade is similar to Grossman and Maggi (2000) and Bougheas and Riezman (2007), and consistent with Bombardini, Gallipoli, and Pupato (2012) who, using data from the International Adult Literacy Survey, show that the skill dispersion has significant effect on trade flows. The present paper complements these studies by considering the impact of trade on skill dispersion; and the results suggest that empirical studies which do not control for the effect of trade on skill acquisition suffer from reverse causality.

This novel aspect of the present model, that trade in turn affects the distribution of human capital, is supported by recent studies. In an interesting paper, Galor and Mountford (2008) argue that opening to trade increases human capital accumulation in the rich countries, but slows it down in poor ones. Using the contemporary data on trade and education, they show that larger trade shares are associated with greater investment in education in OECD economies, but with lower education in developing economies. These findings are consistent with the predictions in Proposition 1.

In another interesting paper, Edmonds, Pavcnik, and Topalova (2010) study the impact of India’s 1991 tariff reform on schooling in the country. They find that schooling decreased in districts where employment concentrated in industries losing high tariff protection. They state that although school tuition in India is free, the costs of books, uniforms, tutoring, and transportation costs combined can be substantial. They argue that reductions in living standards and returns to education are likely channels through which trade liberalization can induce families to take their children out of school. My model is static, and thus does not consider how parents invest in their children’s education. However, in my model, trade lowers the income of individuals working in the manufacturing sector, which in turn induces some individuals in Foreign to become unskilled workers. And this is largely consistent with the finding of Edmonds, Pavcnik, and Topalova (2010).

Finally, using data on Argentinean firms, Bustos (2011) documents skill upgrading after a regional free trade agreement. She finds that the most productive firms serving foreign markets upgrade skill, while the least productive firms

---

10 O’Rourke, Rahman, and Taylor (2007) develop a unified growth model to address how knowledge accumulation and trade have changed the direction of the technical change since the Industrial Revolution. They show that the interplay between technology and trade has played an important role in determining the supply of skilled workers.
serving the domestic market downgrading their skill. In my model, Home’s firms in the manufacturing sector export and upgrade their skill, whereas firms in Foreign’s manufacturing sector do not export and downgrade their skill. These are broadly consistent with Bustos’s findings.

3.2 Welfare analysis

The welfare distribution under free trade is still characterized by eq. [12] with \( \theta^* \) replaced by \( \theta^*_j \). Substituting \( \theta^* \) from [7] into eq. [12] yields

\[
v_j(\theta, p_j^a) = \begin{cases} (p_j^a)^{-\beta} & \text{if } \theta < \theta^a, \\ (p_j^a)^{2-\beta} \theta/2b_j & \text{if } \theta \geq \theta^a. \end{cases}
\]

where the superscript \( a \) denotes autarky. Since \( p_H^a < p < p_F^a \), it then follows that \( (p_H^a)^{-\beta} > p^{-\beta} > (p_F^a)^{-\beta} \). Figure 1(a) and 1(b) shows how trade liberalization affects the distribution of welfare in each country. In each figure, the blue line represents the welfare distribution in autarky. In addition, \( \bar{v}_j^a \) and \( \bar{v}_j \) represent the average entrepreneurial welfare under autarky and free trade, respectively; and these statistics will shortly be used in between-group inequality analysis.

According to Figure 1(a), after trade liberalization individuals with \( \theta > \bar{\theta}_H \) experience an increase in their welfare, whereas individuals with \( \theta < \bar{\theta}_H \) experience a decrease in their welfare. In Foreign, the welfare of all workers with \( \theta \in [1, \bar{\theta}_F] \) increases, but the welfare of workers with \( \theta > \bar{\theta}_F \) falls. Trade has no impact on the welfare of the individual with ability \( \bar{\theta}_j \); i.e., \( v_j(\bar{\theta}_j, p) = v_j(\bar{\theta}_j, p_j^a) \).

Substituting the corresponding equations from [19] into \( v_j(\bar{\theta}_j, p) = v_j(\bar{\theta}_j, p_j^a) \) and using \( p/p_j^a = (\theta^a/\theta^*_j)^{1/2} \) from eq. [7] yields

\[
\bar{\theta}_H = \theta_H^{1/2}(\theta^a)^{\theta/2}, \quad \bar{\theta}_F = \theta_F^{\theta}(\theta^a)^{1-\theta}. \tag{20}
\]

To determine the impact of trade on aggregate welfare, consider the aggregate welfare function given by eq. [13]. The welfare function [13] is convex, and the necessary condition for minimizing \( V_j \) yields the equilibrium condition [14]. Thus, trade is welfare improving. The following proposition summarizes these results.

---

11 Equation [7] implies that \( dp/d\theta^*_j = -p/2\theta^*_j \). Using this in \( dN_j(p, \theta^*_j)/d\theta^*_j = 0 \) yields \( \beta G(\theta^*_j) = (2 - \beta) \int_{\theta^*_j}^{\infty} (\theta/\theta^*)g(\theta)d\theta \), which is the same as the equilibrium condition [14].

12 How much would each individual’s welfare change if she acquires the same level of human capital as in autarky? Consider, for example, Home. Since the supply of skilled workers is fixed, trade will increase the relative price of the manufacturing good, which in turn increases the income of skilled workers. In the present model, the relative price of manufacturing good will
Proposition 2 Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. The only difference between the two countries is their costs of human capital acquisition, and assume that (ceteris paribus) the cost is cheaper in Home (i.e., \( b_H < b_F \)). Moving from autarky to free trade

a. raises (lowers) the welfare of all individuals with \( 1 < \theta < \bar{\theta}_H \) (\( 1 < \theta < \bar{\theta}_F \)), but it lowers (raises) the welfare of all individuals with \( \theta > \bar{\theta}_H \) (\( \theta > \bar{\theta}_F \)) in Home (Foreign).

b. improves the aggregate welfare in both countries.

not increases as much as in the model with static human capital, and increase in human capital through an expansion in intensive margin will make all individuals with \( \theta > \theta^a \) earn more. It then follows from the welfare eq. [3] that the welfare of all individuals with \( \theta \geq \theta^a \) improves. Although some individuals become worse off in the present model, the aggregate welfare is higher than that obtained under the model with static human capital.
Having determined the impact of trade on the welfare distribution, I now turn to investigate the impact of trade on inequality. Before going further, note that the welfare eq. [3] implies
\[
\frac{v_j(p, \theta_1)}{v_j(p, \theta_2)} = \frac{e_j(\theta_1)}{e_j(\theta_2)}
\]
for any two individuals with \(\theta_1\) and \(\theta_2\) in country \(j\). That is, the welfare inequality and income inequality represent the same thing, and therefore in the subsequent discussion I will call them inequality. In investigating the impact of trade on inequality, I consider its impact on between-group and within-group inequalities. Consider first the inequality between skilled and unskilled workers, and following much of the literature on income inequality, I define between-group inequality in country \(j\) as the ratio of the average income of skilled workers (denoted by \(\bar{e}_{sj}\)) to the average income of unskilled workers (denoted by \(\bar{e}_{uj}\)). Note that \(\bar{e}_{uj} = 1\), since any unskilled worker’s wage equals one. The between-group inequality is then given by
\[
\frac{\bar{v}_{sj}}{\bar{v}_{uj}} = \frac{\bar{e}_{sj}}{\bar{e}_{uj}} = \frac{E[\theta \geq \theta^*_j]/\theta^*_j}{1 - G(\theta^*_j)}
\]
The inequality between skilled and unskilled workers in autarky has the same form except \(\theta^*_j\) is replaced by \(\theta^{a}\). Trade increases [decreases] both the aggregate income of skilled workers and their supply in Home [Foreign]. Thus, the net impact of trade on between-group income inequality in each country is ambiguous.

To get a more precise prediction, I assume that ability is (truncated) Pareto distributed: \(G(\theta) = [1 - \theta^{-k}]/[1 - \theta^{-k}_M]\), with shape parameter \(k > 1\). Using a Pareto distribution makes analysis more tractable and has been widely used in trade and productivity literature (e.g., Helpman, Itskhoki, and Redding 2010). In addition, eq. [7] ensures that the corresponding human capital (and thus, firms’ productivity) levels also follow a Pareto distribution. This is consistent with several empirical studies, which have shown that firm sizes follow Pareto distribution (Axtell 2001; Helpman, Melitz, and Yeaple 2004). It is straightforward to show that
\[
\frac{\bar{v}_s}{\bar{v}_u} = \frac{\bar{e}_s}{\bar{e}_u} = \frac{k}{k - 1} \left[ \frac{1 - (\theta^*/\theta^*_M)^{k-1}}{1 - (\theta^*/\theta^*_M)^k} \right],
\]
where for notational simplicity the country index \(j\) is dropped.
Note that if $\theta_M \to \infty$, the distribution becomes standard Pareto; in this case, $\bar{\epsilon}^a_{sj} / \bar{\epsilon}^a_{uj} = \bar{\epsilon}_{sj} / \bar{\epsilon}_{uj} = k/(k - 1)$. Thus, trade has no impact on the inequality between skilled and unskilled workers in either country if ability is standard Pareto distributed. However, if $\theta_M$ is finite, one can show that $\bar{\epsilon}^a_{sH} / \bar{\epsilon}^a_{uH} < \bar{\epsilon}_{sH} / \bar{\epsilon}_{uH}$ and $\bar{\epsilon}^a_{sF} / \bar{\epsilon}^a_{uF} > \bar{\epsilon}_{sF} / \bar{\epsilon}_{uF}$. That is, trade increases [decreases] the inequality between skilled and unskilled workers in skill-abundant Home [Foreign] as shown in Figure 1(a) [Figure 1(b)].

I now turn to analyze the impact of trade on within-group income inequality in each country. Note that there is no variation among the income of unskilled workers, because their income is always equal to one. Following much of the literature, I measure the income inequality among skilled workers by Gini coefficient. Assuming that ability is truncated Pareto distributed, the Gini coefficient for skilled workers in each country is given by (see Appendix B)

$$G_j = 1 + \frac{2}{2k - 1} \left[ k(\theta_j^s / \theta_M) / 1 - (\theta_j^s / \theta_M)^k - (k - 1) / 1 - (\theta_j^s / \theta_M)^{k - 1} \right],$$

where $k > 1$ is the shape parameter and $\theta_M$ is the upper bound of its support. The corresponding coefficient in autarky has the same form except $\theta_j^s$ is replaced by $\theta^a$.

Note that under the standard Pareto distribution (i.e., $\theta_M \to \infty$), the Gini is given by $G_j = 1 / (2k - 1)$; as a result, trade has no impact on the inequality among skilled workers in each country. What happens to the within-group inequality in each country after they trade when $\theta_M$ is finite? Figure 2 shows how the Gini coefficient changes with respect to the ability cutoff $\theta_j^s$ under different shape parameters. It indicates that the impact of trade on the Gini coefficient for skilled workers in each country is ambiguous. The following proposition summarizes these results.

**Proposition 3** Consider two countries, Home and Foreign, with skilled and unskilled workers as described in this section.

a. If ability is standard Pareto distributed, trade has no impact on the between-group and within-group inequality in either country.

c. If ability is truncated Pareto distributed, trade increases [decreases] the between-group inequality in Home [Foreign]; but it has an ambiguous effect on the within-group inequality in each country.

---

13 Differentiating $F(\theta) = [1 - (\theta / \theta_M)^{k - 1}] / [1 - (\theta / \theta_M)^k]$ with respect to $\theta \in (1, \theta_M)$ yields $dF(\theta) / d\theta < 0$ iff $k\theta - \theta^k / \theta_M^{k - 1} - (k - 1)\theta_M < 0$. Note that $k\theta - \theta^k / \theta_M^{k - 1} - (k - 1)\theta_M$ increases with $\theta$, and its supremum is 0; as a result, $dF(\theta) / d\theta < 0$. 

---

1081
The above results are different from the Stolper-Samuelson theorem in the standard HO model in two important ways. First, if the human capital endowment in each country had not changed after the trade liberalization, the income inequality between skilled and unskilled workers in Home would have increased by the amount predicted by the Stolper-Samuelson theorem. This effect is represented by moving from point A to B in Figure 1(a). However, in the present model, trade changes the human capital in each country both in extensive and intensive margins which in turn put a downward pressure on the inequality between these groups. In this case, the average entrepreneurial income in Home will move from point B to C in Figure 1(a). Indeed, according to Proposition 2, where ability is distributed standard Pareto, this downward pressure exactly cancels out the Stolper-Samuelson effect. The corresponding effects in Foreign are represented by moving from A to B to C in Figure 1(b). In sum, the impact

---

14 In this case, $\bar{e}_s/\bar{e}_u = (2p/p_H^a - 1)\bar{e}_s^a/\bar{e}_u^a$, where $p$ is the relative price of the manufacturing good after trade (detailed analysis available upon request). Since $p > p_H^a$, the coefficient $(2p/p_H^a - 1) > 1$ captures the Stolper–Samuelson effect.
of trade on between-group inequality in the present model is not as strong as that in the standard HO model.

Second, since in the HO model factor endowments are fixed and all skilled workers have the same level of human capital, the model is not suitable for analyzing the impact of trade on within-group inequality. In the present model, however, factor endowments are endogenously determined, and the income of skilled workers is proportional to the level of human capital they acquired. As a result, one can analyze the impact of trade on within-group inequality.

What empirical implications can one draw from the present model? The wage gap between skilled and unskilled workers has dramatically increased since the early 1980s in many developed countries and especially in the U.S. This dramatic change in the wage structure combined with increased globalization with developing countries during the same period led some economists to conclude that trade with developing countries accounts for a large share of the inequality (Wood 1998). However, the present model suggests that trade cannot be a driving factor behind the growth in the skill-premium. In addition, the skill-premium has increased in many developing countries after they opened to trade in the 1980s and 1990s (Goldberg and Pavcnik 2007). Finally, the present model predicts that the impact of trade on the wage dispersion among skilled workers is ambiguous, which suggests that trade may not be the driving factor behind the increased wage dispersion within occupations and sectors observed in many countries (Goldberg and Pavcnik 2007).

3.3 Political-economy equilibrium

Although trade increases aggregate welfare, not all agents gain from the free trade. Is trade a viable option? To answer this question, I make two further

---

15 In my model, markets are perfectly competitive, and there is only one-way flow of each good between countries. Epifani and Gancia (2008) and Unel (2010) develop two-sector, monopolistic competition trade models where trade quantitatively has a significant impact on the skill premium.

16 Dinopoulos and Unel (2014) introduce human capital formation presented in this paper into a trade model with monopolistic competition to study the impact of trade on income distribution. They find that trade liberalization disproportionately benefits the most talented individuals who manage exporting firms. Although countries are symmetric in their model, the intuition behind their mechanism suggests that the results would remain the same in a North-South framework. Ma (2014) develops a monopolistic competition model of trade with occupational choice where exporting and FDI have a significant impact on the income dispersion among individuals.
assumptions. First, I assume that a country follows a free trade policy if the median voter prefers free trade to autarky. In other words, a country chooses to trade if the majority of its population approves it.\(^1\) Second, I assume that ability follows the following Pareto distribution:

\[
G(\theta) = \frac{1}{\theta^k} \frac{1}{(\theta^k)^{(1-k)}}
\]

where \(k > 1\) is the shape parameter.\(^2\)

Under this distribution function, the aggregate welfare function then becomes,

\[
V_j = \left(2b_j\right)^{\frac{x_j}{\alpha}} \left[\theta_j^{\alpha} + \frac{\theta_j^{\alpha-k}}{k-1}\right],
\]

and attains its (global) minimum at\(^3\)

\[
\theta^a = \left[\frac{2k - \beta}{\beta(k-1)}\right]^{\frac{1}{k}}
\]

Using eqs [7] and [17] together with the Pareto distribution, it is straightforward to show that under free trade the ability cutoffs are given by

\[
\theta_H^* = \left[\frac{1 + b_H^k}{2b^k}\right]^{\frac{1}{\theta^a}}, \quad \theta_F^* = b\theta_H^*, \quad b = \frac{b_F}{b_H},
\]

where \(\theta^a\) is given by eq. [22]. Note that \(\theta_H^*\) decreases in \(b\), whereas \(\theta_F^*\) increases in it.

Since all Home workers with \(\theta > \tilde{\theta}_H\) benefit from free trade (see Figure 1.a), Home chooses to trade under majority voting if and only if \(G(\tilde{\theta}_H) < 0.5\). Equations [20] and [23] then yield

\[
G(\tilde{\theta}_H) < 0.5 \iff b^{-k} < \gamma_H, \quad \gamma_H = 2\left[\frac{2\beta(k-1)}{2k - \beta}\right]^{\frac{1}{\alpha}} - 1.
\]

Observe that \(2\beta(k-1)/(2k - \beta) < 1\), and the latter inequality implies that \(\gamma_H < 1\). If \(\gamma_H < 0\), then there is no \(b \in \mathbb{R}_+\) that satisfies the condition \(b^{-k} < \gamma_H\). In this

---

\(^1\) Mayer (1984) is the first seminal work that rigorously investigates the role of majority voting in determining trade policy. In particular, he analyzes how factor-ownership distribution and voter participation costs under majority voting affects a country’s tariff policy.

\(^2\) I consider the standard Pareto distribution, because there is no closed form solutions to ability cutoffs under the truncated Pareto distribution.

\(^3\) This directly follows from Proposition 3. Alternatively, differentiating eq. [21] with respect to \(\theta_j^*\) also yields eq. [22]. Note that second-order condition is satisfied, since \(d^2V/d\theta_j^{\alpha-k} = k\beta(2b_j)^{-\alpha/(\alpha-k)}(\theta_j^\alpha)^{\alpha-k}/2 > 0\). Furthermore, one can show that \(dV/d\theta_j^* < 0\) for \(\theta_j^* < \theta^a\) and \(dV/d\theta_j^* > 0\) for \(\theta_j^* > \theta^a\).
case, Home chooses not to trade. Note that $\gamma_H$ is always negative whenever $\beta \leq 0.5$. In addition, it increases with $k$. Thus, the condition $\gamma_H > 0$ holds when $\beta$ and $k$ are sufficiently high. In this case, Home chooses to trade if $b > \gamma_H^{-1/k}$.

Foreign, on the other hand, chooses to trade under majority voting if and only if $G(\theta_F) > 0.5$. Following the same steps as in the previous case yields

$$G(\theta_F) > 0.5 \iff b^k > \gamma_F = \frac{2(1 - \beta) + \beta^k}{2k - \beta} - 1.$$  

Since $2\beta(k - 1)/(2k - \beta) < 1$, it then follows that $\gamma_F < 1$. Thus, the condition $b > \gamma_F$ always holds. As a result, Foreign always chooses to trade.

**Proposition 4** Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. Suppose that $b = b_F/b_H > 1$ so that (ceteris paribus) the cost of human capital formation is cheaper in Home. Suppose further that each country chooses to trade if the majority of its population approves it. These countries choose to trade if and only if $\gamma_H > 0$ and $b > \gamma_H^{-1/k}$, where $\gamma_H$ is given by eq. [24].

The reason that Foreign always chooses to trade crucially depends on the assumption that ability follow a Pareto distribution. Although Pareto distribution is a heavy-tailed, it is skewed more towards left. Using this distributional assumption, it is easy to show that $G(\theta^a) > 1/2$, i.e. more than half of the population in each country choose to become unskilled workers in autarky. Since trade expands the fraction of unskilled workers in Foreign and these individuals are better off under free trade, it then follows that Foreign always chooses to trade.

The above proposition implies that an egalitarian government (in the sense that it gives equal voting rights to everyone) in Home will not choose to trade unless the share of manufacturing in total expenditure is sufficiently high and the cost of human capital acquisition is sufficiently low (compared to that in Foreign), because the majority of workers will be worse off. On the other hand, an egalitarian government in unskilled labor abundant Foreign will always choose to trade, because the majority of workers in Foreign will be better off under free trade. These results are broadly consistent with Dutt and Mitra (2005) who show that left-wing governments in capital-abundant countries adopt more protectionist trade policies than right-wing ones, and left-wing governments in labor-abundant countries adopt more pro-trade policies than right-wing ones.

---

20 For example, $\beta = 0.8$ and $k = 4$ implies that $\gamma_H \approx 0.018$. Thus, based on majority voting, Home chooses to trade if $b > 2.75$.

21 Substituting $\theta^a$ from eq. [22] into $G(\theta) = 1 - \theta^{-k}$ yields $G(\theta^a) = (2k - \beta k)/(2k - \beta) > 1/2$, and the latter inequality holds because $2k > \beta(2k - 1)$.  


3.4 Change in the cost of human capital acquisition

The analysis so far has focused on how trade liberalization affects allocation of skills across sectors and its effects on income distribution and welfare in each country. Another interesting question to address is the implication of a change in the cost parameter $b$ in one country on the income distribution and welfare in each country when countries freely trade with each other. The following lemma characterizes the impact of this change on the ability cutoff level in each country (see Appendix C for the proof).

**Lemma 3** A reduction in $b_j$ decreases the ability cutoff $\theta_j^*$ in country $j$, while increasing it in the other country.

Without loss of generality, consider a reduction in Foreign’s cost parameter $b_F$.\(^{22}\) According to the above lemma, such a reduction decreases the ability cutoff level $\theta_F^*$ in Foreign, while increasing $\theta_H^*$ in Home. Thus, the fraction of individuals who acquire human capital increases (decreases) in Foreign (Home). Furthermore, eqs [5] and [7] indicate that each worker in manufacturing in Foreign will acquire more human capital. The opposite will be observed in Home. Assuming that ability is truncated Pareto distributed, the income inequality between skilled and unskilled workers increases in Foreign and decreases in Home. As shown in Appendix C, such a reduction in $b_F$ improves Foreign welfare and reduces Home welfare. Appendix C also shows that a reduction in $b_H$ (instead of $b_F$) improves welfare in both countries. The following proposition summarizes the results assuming that such a reduction happens in Foreign.

**Proposition 5** Consider two freely trading countries (Home and Foreign), each producing agricultural and manufacturing goods. A reduction in the cost of human capital (in the form of lowering $b_F$) in Foreign increases human capital both at extensive and intensive margins, and improves welfare in Foreign, while having completely opposite effects on these variables in Home. In addition, assuming that ability is distributed truncated Pareto, lowering $b_F$ increases [decreases] the income inequality between skilled and unskilled workers in Foreign [Home], and

---

\(^{22}\) How does a change in the dispersion parameter $k$ in Pareto distribution in one country affect each economy? Note that in my model the distributions of talent across countries are the same. Introducing asymmetry in the dispersion parameter makes distributions differ across countries. How does a reduction in $k_F$, for example, affect each economy. My analysis (available upon request) shows that with $k_F < k_H, \text{Foreign may have the comparative advantage in manufacturing even though } b_H < b_F$. In this case, trade enhances human capital in Foreign both at extensive and intensive margins.
has an ambiguous effect on the income inequality among skilled workers in each country.

A reduction in $b_F$ makes human capital formation more affordable in Foreign, which induces more people to work in manufacturing, i.e. the ability cutoff necessary to work in manufacturing falls in Foreign. This will increase individual-level human capital as well as the income per capita for workers in Foreign’s manufacturing. An expansion in Foreign’s manufacturing sector lowers relative price of the manufacturing good as well. This makes human capital formation more costly for some workers in Home, and thus they prefer to work in agricultural sector. In this case, the ability cutoff $\theta_H$ will raise in Home.

A reduction in the ability cutoff in Foreign increases each manufacturing worker’s income (see eq. [10]), which increases the aggregate income in Foreign. This, combined with a reduction in the price of manufacturing, improves the aggregate welfare in Foreign. On the other hand, a fall in relative price of the manufacturing good combined with a lower human capital reduces each manufacturing worker’s income in Home. Consequently, Home aggregate income decreases. Although reduction in the price of the manufacturing good has a positive effect on welfare, reduction in aggregate income overcomes this effect, and thus, aggregate welfare decreases in Home.

In the case of a reduction in the cost of human capital in Home (i.e., $b_H$), Home welfare improves since relative price of the manufacturing good decreases and the aggregate expenditure increases. Although such a reduction decreases the aggregate expenditure in Foreign, reduction in the relative price of manufacturing dominates the latter, and thus improves welfare in Foreign as well.

4 Concluding remarks

This paper has studied the interaction between human capital formation and trade using a two-sector, two-country model of trade in which individuals choose to become either an unskilled worker and work in agriculture or skilled worker and work in manufacturing. The only difference between the two countries is that the cost of human capital acquisition is lower in Home. The analysis shows that this difference determines the pattern of comparative advantage across countries. However, trade also affects the distribution of human capital in each country.

The present model can be extended in several direction. Dinopoulos and Unel (2013), for example, incorporate labor market rigidities into the present model to study how trade affects income distribution and unemployment.
Another extension is to introduce a tariff and investigate the income inequality and welfare implications of a government policy that uses the revenue from tariffs to make human capital formation more affordable for workers. Using this extended framework to analyze how the educational policy determined by different legislative systems (e.g., social planner vs. majority voting) may uncover interesting results. The model assumes a well-functioning financial market for individuals who wish to acquire human capital. Incorporating credit market imperfections in financing the cost of human capital formation is another extension.

Appendix

A. Proof of Lemma 3

Without loss of a generality, suppose that there is a reduction in $b_F$. Totally differentiating eq. [17] with respect to $b_F$ yields

$$\sum_j A_j \frac{d\theta_j^*}{db_F} = 0, \quad A_j = g(\theta_j^*) + 2 - \beta \frac{\mathbb{E}[\theta \in [\theta_j^*]]}{\theta_j^*^2}. \tag{25}$$

Differentiating eq. [7] with respect to $b_F$, on the other hand, yields

$$\frac{d\theta_H^*}{db_F} = -\frac{2\theta_H^*}{p} \frac{dp}{db_F}, \quad \frac{d\theta_F^*}{db_F} = \frac{2}{p^2} - \frac{2\theta_F^*}{p} \frac{dp}{db_F}. \tag{26}$$

Substituting these into eq. [25] and rearranging terms yields

$$\frac{dp}{db_F} = \frac{A_F}{p \sum_j \theta_j^* A_j} > 0. \tag{27}$$

It then follows from eq. [26] that $d\theta_H^*/db_F < 0$. Substituting eq. [27] into the second equation in eq. [26], on the other hand, yields

$$\frac{d\theta_F^*}{db_F} = \frac{2\theta_H^* A_H}{p^2(\theta_H^* A_H + \theta_F^* A_F)} > 0. \tag{28}$$

B. Gini coefficient

Let $E_{sj}(\bar{\theta})$ denote the total income of all skilled workers whose ability level is less than or equal to $\bar{\theta}$, and $E_{sj}$ denote the total income of all skilled workers in
country $j$. If ability is Pareto distributed, truncated over the support $[1, \theta_M]$, it then follows that

$$
\frac{E_s(\bar{\theta})}{E_s} = \frac{\int_{\theta_0}^{\theta} e(\theta) g(\theta) d\theta}{\int_{\theta_0}^{\theta_M} e(\theta) g(\theta) d\theta} = \frac{1 - (\theta^* / \bar{\theta})^{k-1}}{1 - (\theta^* / \theta_M)^{k-1}},
$$

where the country index $j$ is dropped for notational simplicity.

Let $x$ denote the fraction of skilled workers whose ability is less than or equal to $\theta^*$. Thus,

$$
x = \frac{G(\theta^*) - G(\theta^*)}{1 - G(\theta^*)} = \frac{1 - (\theta^* / \bar{\theta})^{k}}{1 - (\theta^* / \theta_M)^{k}},
$$

and note that $x \in [0, 1]$. Solving this equation for $\theta^*/\bar{\theta}$ yields

$$
\frac{\theta^*}{\bar{\theta}} = \left[ 1 - \left( \frac{\theta^*}{\theta_M} \right)^{k} \right]^{\frac{1}{k}}.
$$

Substituting $\theta^*/\bar{\theta}$ from eq. [30] into eq. [29] yields the following Lorenz curve:

$$
\mathcal{L}(x) = \frac{1 - \left[ 1 - \left( \frac{\theta^*}{\theta_M} \right)^{k} \right] x^{\frac{k-1}{k}}}{1 - (\theta^* / \theta_M)^{k-1}}.
$$

Finally, the Gini coefficient is given by

$$
G = 1 - 2 \int_{0}^{1} \mathcal{L}(x) dx.
$$

Substituting $\mathcal{L}(x)$ into the above equation and integrating over $[0, 1]$ yields the equation in the main text.

### C. Proof of Proposition 5

Without loss of generality, consider again a reduction in $b_F$. Since this reduction lowers $p$, eq. [5] immediately implies that $h$ will decrease in Home. Totally differentiating eq. [5] with respect to $b_F$ yields

$$
\frac{dh_F}{db_F} = \frac{\theta}{b_F} \frac{dp}{db_F} - \frac{\theta p}{b_F^2}.
$$

Substituting eq. [27] into the above equation and using eq. [7] yields
\[
\frac{dh_F}{db_F} = \frac{\theta}{b_F p} \left[ \frac{1}{\theta_H A_H/A_F + \theta_F^* B} \right] < 0,
\]
where \( A_j \) is given by eq. [25].

To determine the welfare impact, I use the aggregate welfare function 
\( \forall_j = p^{-\beta} E_j \), where \( E_j \) is given by eq. [11]. Differentiating eq. [11] with respect to 
\( b_F \) and using eq. [26] yields
\[
\frac{dV_H}{db_F} = -\beta p^{-1-\beta} \left[ G(\theta_H^*) - \left( \frac{2 - \beta}{\beta} \right) \frac{B[\theta \geq \theta_H^*]}{\theta_H^*} \right] \frac{dp}{db_F} > 0, \tag{33a}
\]
\[
\frac{dV_F}{db_F} = -\beta p^{-1-\beta} \left[ G(\theta_F^*) + \left( \frac{2 \theta_H^* A_H}{\beta \theta_H^* A_F} \right) \frac{B[\theta \geq \theta_F^*]}{\theta_F^*} \right] \frac{dp}{db_F} < 0. \tag{33b}
\]
where \( dp/db_F > 0 \). The expression in square brackets in eq. [33a] is negative since Home exports manufacturing good as indicated in eq. [18]. As result, 
\( dV_H/db_F > 0 \).

However, if there is a reduction in \( b_H \), following the same steps yields
\[
\frac{dV_H}{db_H} = -\beta p^{-1-\beta} \left[ G(\theta_H^*) + \left( \frac{1 + 2 \theta_H^* A_H}{\theta_H^* A_H} \right) \frac{B[\theta \geq \theta_H^*]}{\theta_H^*} \right] \frac{dp}{db_H} < 0, \tag{34a}
\]
\[
\frac{dV_F}{db_H} = -\beta p^{-1-\beta} \left[ G(\theta_F^*) - \left( \frac{2 - \beta}{\beta} \right) \frac{B[\theta \geq \theta_F^*]}{\theta_F^*} \right] \frac{dp}{db_H} < 0, \tag{34b}
\]
where \( dp/db_H > 0 \). The expression in square brackets in eq. [34b] is now positive since Foreign exports agricultural good, and thus 
\( dV_F/db_H < 0 \).

Acknowledgments: I am indebted to Shankha Chakraborty, Elias Dinopoulos, and Yoto Yotov for their valuable comments and suggestions. I also thank two anonymous referees and the editor, Till Requate, for their very helpful comments.

References


